INTERNATIONAL JOURNAL FOR NUMERICAL AND ANALYTICAL METHODS IN GEOMECHANICS, VOL. 22, 229–238 (1998)

# SHORT COMMUNICATION

# FIELD STRESSES AROUND AN INCLINED SHAFT

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## SUMMARY

In this study, the vertical overburden pressure in the vicinity of an inclined circular underground opening is defined by relation to the geometry of the medium. The stresses around the opening consisting of six components are induced by geostatic field stress. In this regard, the inclined circular opening i.e. shaft changes the original stress condition and an interrupted region develops beneith it. Three zones are defined on a plane which is perpendicular to the axis of shaft. The mathematical expressions of vertical overburden stresses at these three zones are established, respectively. The example given in appendix demonstrates the variation of radial and tangential stresses around the inclined shaft. It is seen that the stress distributions in the third zone, which includes the interrupted region, beneith the shaft display different configurations than that of those obtained by undisturbed field stresses. In the interrupted region the stresses around the shaft linearly grow up due to increasing overburden pressure by radial distance from the periphery of the shaft. At the boundary of interrupted region stresses jump to the original values induced by field stresses. © 1998 John Wiley & Sons, Ltd.

Int. J. Numer. Anal. Meth. Geomech., Vol. 22, 229–238 (1998)

Key words: inclined shaft; stress distribution; field stress

## INTRODUCTION

The stress distribution around circular opening is frequently mentioned in literature. The previous studies are mostly concerned with the stress distribution around vertical shafts in-plane strain condition.[1,2] As for the inclined shafts, very little literature has been found. Some Japanese researchers carried out three-dimensional stress distribution analysis around inclined circular shafts. One example for these is the study carried out by Hiamatsu and Yukitoshi.[3] These researchers provided the general expressions for triaxial stresses around an inclined shaft with field stresses in cartesian co-ordinate system oriented in any direction. When the geostatic field stress is taken as the prime source, the stress around shaft should be interpreted by expressions briefly defined by geostatic field stress and geometry of the medium. This study particularly incorporates corresponding parameters, namely, the vertical field stress $\sigma_z$, horizontal to vertical field stress ratio $K$ and the inclination angle $\alpha$ of shaft axis with respect to vertical one in a more definite manner.

Furthermore, for the time being no attention is attached to the mutual relationship between the geometry of the medium and vertical stress. An inclined shaft forms barrier to the vertical stress

CCC 0363–0961/98/030229–10$17.50

*Received 10 September 1996*
*Revised 21 May 1997*

field forming an interrupted zone beneith the inclined shaft. Consequently, the induced stresses under the shaft are expected to be rebuilt due to this interruption. Thus, the scope of this study covers the definition of field stresses with respect to the geometry of the medium and establishment of related equations.

## STRESSES AROUND AN INCLINED SHAFT

The original field stresses are vertical $\sigma_z$ and horizontal $K\sigma_z$ stresses. The variation of induced stresses is defined in a plane $P$ that is perpendicular to the axis of the shaft (Figure 1). The shaft is inclined to vertical by an angle of $\alpha$, so the plane $P$ is inclined to horizontal by this angle. The origin, initial line and $z$-axis of cylindrical co-ordinates $(r, \theta', \zeta)$ coincide with point O, the $y', y'$-axis and the axis of shaft, respectively. The state of stress at any point in $P$-plane is defined by six components of stresses given as below,[4–6]

$$\sigma_r = \frac{1}{2}(K\sigma_z \cos^2\alpha + \sigma_z \sin^2\alpha + K\sigma_z)\left(1 - \frac{a^2}{r^2}\right)$$
$$+ \frac{1}{2}(K\sigma_z \cos^2\alpha + \sigma_z \sin^2\alpha\sigma_z + K\sigma_z)\left(1 + \frac{3a^4}{r^4} - \frac{4a^2}{r^2}\right)\cos 2\theta' \quad (1)$$

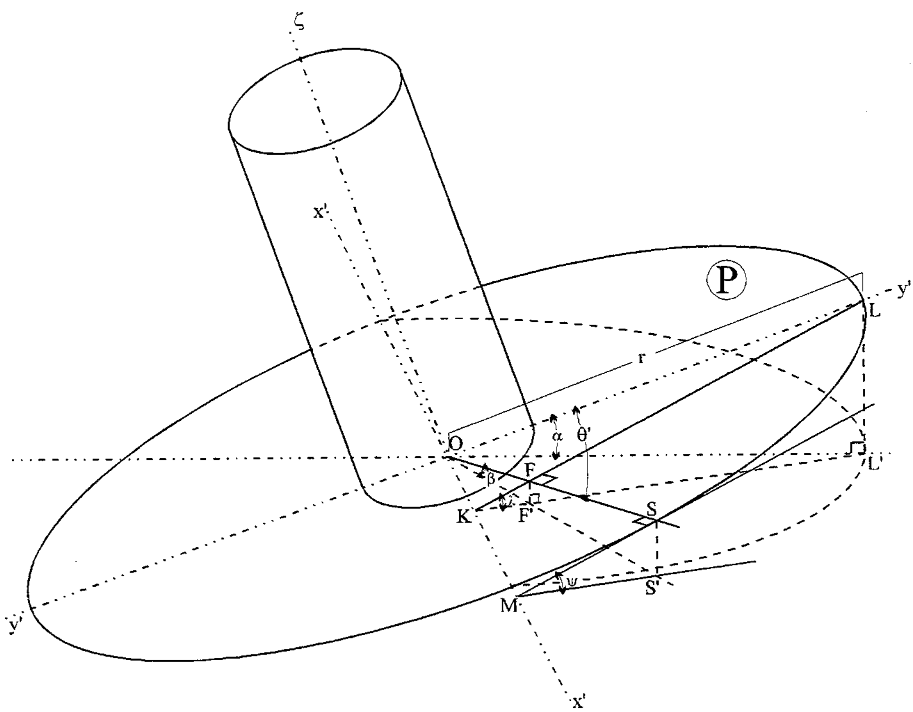



Figure 1. The geometry of the problem

$$\sigma_{\theta'} = \frac{1}{2}(K\sigma_z \cos^2\alpha + \sigma_z \sin^2\alpha + K\sigma_z)\left(1 + \frac{a^2}{r^2}\right) - \frac{1}{2}(K\sigma_z \cos^2\alpha + K\sigma_z \sin^2\alpha - K\sigma_z)$$
$$\times\left(1 + \frac{3a^2}{r^2}\right)\cos 2\theta' \quad (2)$$

$$\tau_{r\theta'} = \frac{1}{2}(K\sigma_z \cos^2\alpha + \sigma_z \sin^2\alpha - K\sigma_z)\left(1 - \frac{3a^4}{r^4} + \frac{2a^2}{r^2}\right)\sin 2\theta' \quad (3)$$

$$\sigma_\zeta = \sigma_z \cos^2\alpha + K\sigma_z \sin^2\alpha - 2v(K\sigma_z \cos^2\alpha + K\sigma_z \sin^2\alpha - K\sigma_z)\left(\frac{a^2}{r^2}\right)\cos 2\theta' \quad (4)$$

$$\tau_{r\zeta} = (\sigma_z - K\sigma_z)\cos\alpha \sin\alpha \cos\theta'\left(1 - \frac{a^2}{r^2}\right) \quad (5)$$

$$\tau_{\theta'\zeta} = (\sigma_z - K\sigma_z)\cos\alpha \sin\alpha \sin\theta'\left(1 + \frac{a^2}{r^2}\right) \quad (6)$$

where, $\sigma_r$, $\sigma_\theta$ and $\sigma_\zeta$ denote the radial, tangential and vertical stresses, respectively and $\tau_{r\theta}$, $\tau_{r\zeta}$ and $\tau_{\theta\zeta}$, denote the shearing stresses. The parameters $a, r, \theta'$ and $v$ define the radius of shaft, radial distance from origin, positioning angle (Figure 1) and Poisson's ratio, respectively.

## THE FIELD STRESSES

The field stresses around an inclined shaft are investigated with the aid of Figure 2. The vertical stress from surface to point O is denoted by $\sigma_{zo}$. Three zones exist on the $P$-plane. First zone (1) takes part at the upper half of the plane with respect to the shaft inclination. Second zone (2) extends along the horizontal axis $xx$, and the third zone (3) is the lower half of the $P$-plane. In the first zone, the vertical stress field $\sigma_{zo}$, reduced by radial distance $r$ from centre O, due to the decreasing depth. The reduction of depth on the $P$-plane by increasing radial distance $r$ from centre O can be defined by using the trigonometric relationship. In Figure 1, the angle $\beta$ defines the slope angle of the radial trajectory OS. This slope angle of $\beta$ reduces from its maximum $\alpha$ to zero away from O$y'$ to O$x'$. The relationship of this angle with the inclination angle of $\alpha$ and position angle of $\theta'$ can be found by the aid of drawings seen in Figure 1. The perpendicular line from the point L to the radial line OS, intersects OS at the point F. Two triangles which are LKL$'$ and SOS$'$ intersect along the piece of line FF$'$. From Figure 1,

$$\sin\beta = \frac{\mathrm{FF'}}{\mathrm{OF}} \quad (7)$$

$$\mathrm{FF'} = \mathrm{KF}\sin\lambda \quad (8)$$

$$\sin\lambda = \frac{\mathrm{LL'}}{\mathrm{KL}} \quad (9)$$

$$\mathrm{LL'} = r\sin\alpha \quad (10)$$

$$\mathrm{KL} = \frac{\mathrm{OL}}{\cos(90^\circ - \theta')} = \frac{r}{\sin\theta'} \quad (11)$$

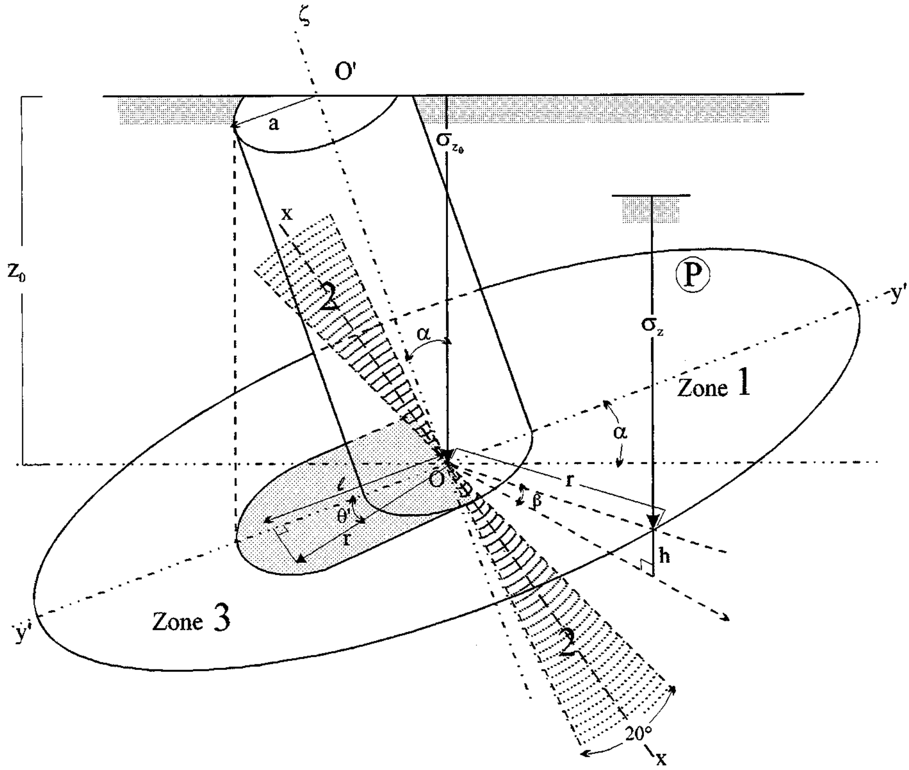



Figure 2. The zones on $P$-plane around the shaft

Thus,

$$\sin\lambda = \sin\alpha\sin\theta' \tag{12}$$

$$\mathrm{KL} = \mathrm{KL} - \mathrm{LF} \tag{13}$$

$$\mathrm{LF} = r\sin\theta' \tag{14}$$

$$\mathrm{KF} = r\cdot\left(\frac{1}{\sin\theta'} - \sin\theta'\right) \tag{15}$$

By (8), (12) and (15) the following is obtained:

$$\mathrm{FF}' = r\sin\alpha(1 - \sin^2\theta') = r\sin\alpha\cos^2\theta' \tag{16}$$

OF is expressed as

$$\mathrm{OF} = r\cos\theta' \tag{17}$$

The substitutions of (16) and (17) into (7) lead to

$$\sin\beta = \sin\alpha\cos\theta' \tag{18}$$

and

$$\cos\beta = (1 - \sin^2\alpha \cos^2\theta')^{1/2} \tag{19}$$

Returning to the field stress, from Figure 2, the field stress at a point $P$ at a radial distance $r$ in a direction defined by departure angle $\theta'$ is

$$\sigma_z = \sigma_{zo} - h\rho g \tag{20}$$

By the relationship $h = r\sin\beta$,

$$\sigma_z = \sigma_{zo} - \rho g r \sin\beta \tag{21}$$

By considering (18), the vertical field stress is defined as

$$\sigma_z = \sigma_{zo} - \rho g r \sin\alpha \sin\theta' \tag{22}$$

where $\rho g$ is unit weight of rock, $\rho$ is the density and $g$ is the gravitational acceleration.

The second zone extends along the $xx$ axis. Considering the inclination induces negligible effect on the depth from surface at short distances along the $xx$ axis, the area swept by $20^\circ$ angle is defined as the second zone (Figure 2).

In the third zone, the field stress grows by increasing depth and two regions are recognized. The first region lies under the inclined shaft. This interrupted region takes the shape of the image of the shaft on $P$-plane. The shape of this image is such that the two sides are parallel to each other and joined at ends by a semi-circular border (Figure 6). If a point in a direction deviated from the $yy'$ axis by an angle $\theta'$, and at a radial distance $r$ is taken, for this point to be in the interrupted region, the following conditions must be satisfied:

$$r\sin\theta' < a \quad \text{and} \quad r\cos\theta' < l \tag{23}$$

where $a$ is the radius of the shaft and $l$ indicates the distance between the projection of point O′ on $P$-plane and origin O. Since the lower border of region encloses a semi-circle with radius $a$, the following conditions should be satisfied in order to keep the point inside the border:

$$l < r\cos\theta' < l + a \tag{24}$$

and

$$r^2 + l^2 - 2rl\cos\theta' < a \tag{25}$$

Vertical field stress beneith the shaft is calculated using geometrical expressions shown in Figure 3. A vertical plane passing through the point $P$ is considered and this plane makes a vertical elliptic cross-section $E$, with the shaft. The parameters of this ellipse are

$$a' = \frac{a}{\sin\alpha} \quad \text{and} \quad b = a \tag{26}$$

where $a$ is the radius of shaft

The abscissa of the point $P''$ on the ellipse $E$ is,

$$x = \left[\left(1 - \frac{y^2}{b^2}\right) a'^2\right]^{1/2} \tag{27}$$

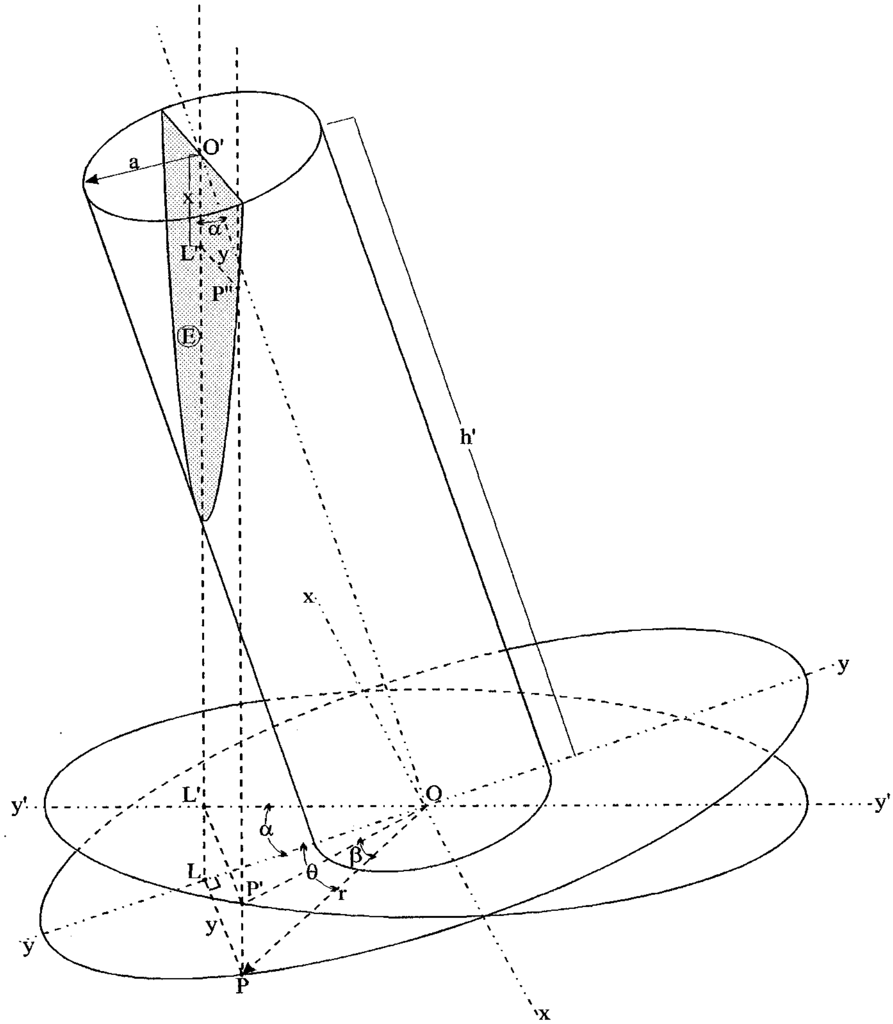



Figure 3. The vertical field pressure distance PP″ on the point $P$ in interrupted region Upon substitution of (26) into (27) the following is obtained

From Figure 3, $y = r\cos\theta'$

$$x = \left(\frac{a^2 - r^2\sin^2\theta'}{\sin^2\alpha}\right)^{1/2} \tag{28}$$

By Figure 3, the vertical distance PP″ is,

$$\mathrm{PP}'' = \mathrm{LL}'' = \mathrm{O'L} - x \tag{29}$$

$$\mathrm{O'L} = \frac{h'}{\cos\alpha}$$

and

$$h' = r\cos\theta' \cot\alpha$$

Therefore,

$$\mathrm{O'L} = r\frac{\cos\theta'}{\sin\alpha} \tag{30}$$

By (28)–(30) the vertical distance PP$''$ is found by

$$\mathrm{PP}'' = \frac{r\cos\theta'}{\sin\alpha} - \left(\frac{a^2 - r^2\sin^2\theta'}{\sin^2\alpha}\right)^{1/2} \tag{31}$$

where $a$ is the radius of shaft. Therefore, the field stress on the point $P$ is,

$$\sigma_z = \left[\frac{r\cos\theta'}{\sin\alpha} - \left(\frac{a^2 - r^2\sin^2\theta'}{\sin^2\alpha}\right)^{1/2}\right]\rho g \tag{32}$$

For the points beyond the interrupted zone, the field stress grows by increasing depth. Hence, by the similar approach made for the first zone, the vertical field stresses is obtained by

$$\sigma_z = \sigma_{zo} + pgr\sin\beta \quad \text{or} \quad \sigma_z = \sigma_{zo} + pgr\sin\alpha\cos\theta' \tag{33}$$

## CONCLUSIONS

In this paper, the geometry of the medium is found to be a significant factor in determining the field stresses around an inclined shaft. In this respect, three zones are defined on a plane perpendicular to the inclined shaft axis. Mathematical expressions defining the vertical field stresses at these zones are established, respectively.

The influence of inclination is found to be negligible on the overburden height and consequently on the variation of stresses tin the vicinity of shaft. But, the situation is completely different for the zone beneith the inclined shaft where the vertical overburden pressure is obstacled by shaft. In this region which is called as interrupted region, the stresses linearly grow up from the periphery of the shaft due to the linearly increasing depth of overburden. These stresses jump to their values imposed by undisturbed overburden pressure at the boundary of interrupted–uninterrupted regions in third zone.

## APPENDIX

In this example, by taking the radius of shaft as $a$, and radial distance as $r$ the radial $\sigma_r$ and tangential $\sigma_{\theta'}$ stresses are calculated depending on $r/a$.

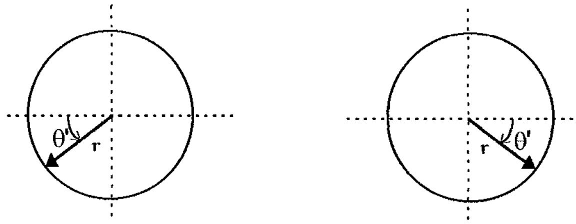
The inputs are

$a$: Radius of shaft; 3 m

$h$: The depth of the centre of shaft from surface; 200 m

$K$: Earth pressure ratio; 0·5

$\alpha$: Shaft inclination with respect to vertical




The departure in 3rd. zone    The departure in 1st. zone

The results are given in the followings

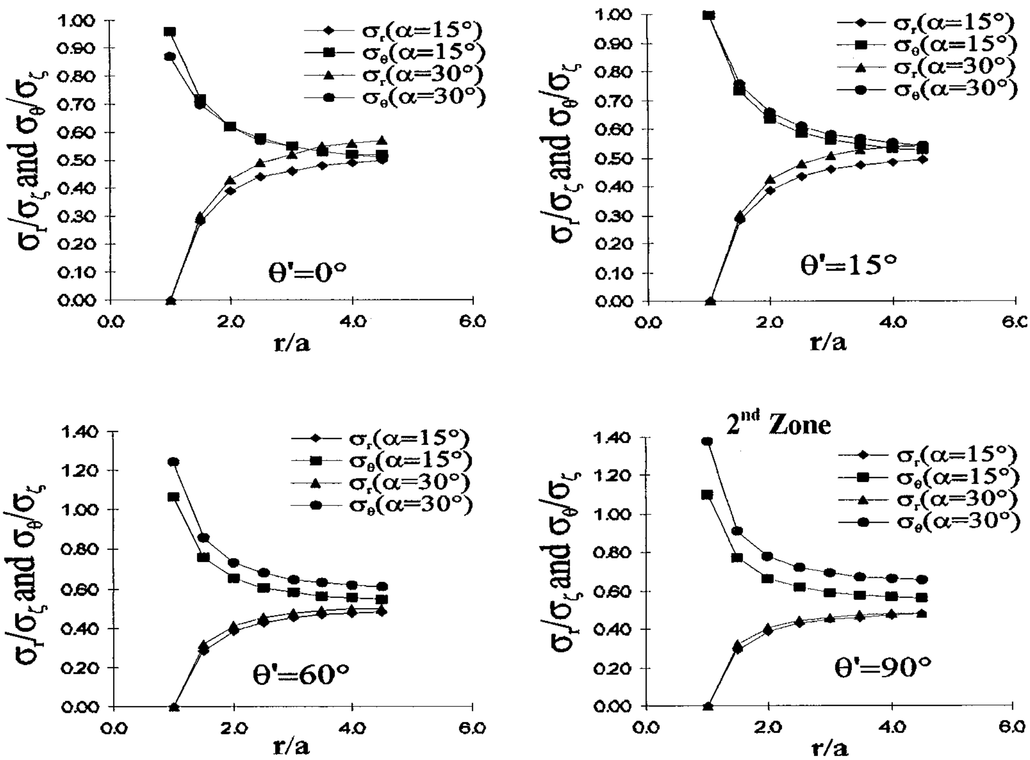



Figure 4. The variation of normalised radial $\sigma_r$ and tangential $\sigma_{\theta'}$ stresses with respect to $r/a$ in 1st zone

## DISCUSSIONS

Figures 4 and 5 are well explanatory for the radial $\sigma_r$ and tangential $\sigma_\theta$ stress distributions around an inclined shaft. The stresses are normalized with respect to the vertical stress $\sigma_\zeta$ acting on the plane $P$ (Figure 2). The stress distributions in the first zone where the field stress is uniformly distributed over the entire area, are regular as expected. In this zone, along $\theta' = 0$ direction, the radial stress $\sigma_r$ distribution on the plane with inclination angle of $\alpha = 15°$ follows a path under the one obtained on the plane with inclination angle of $\alpha = 30°$. By increasing departure from $\theta'$ from

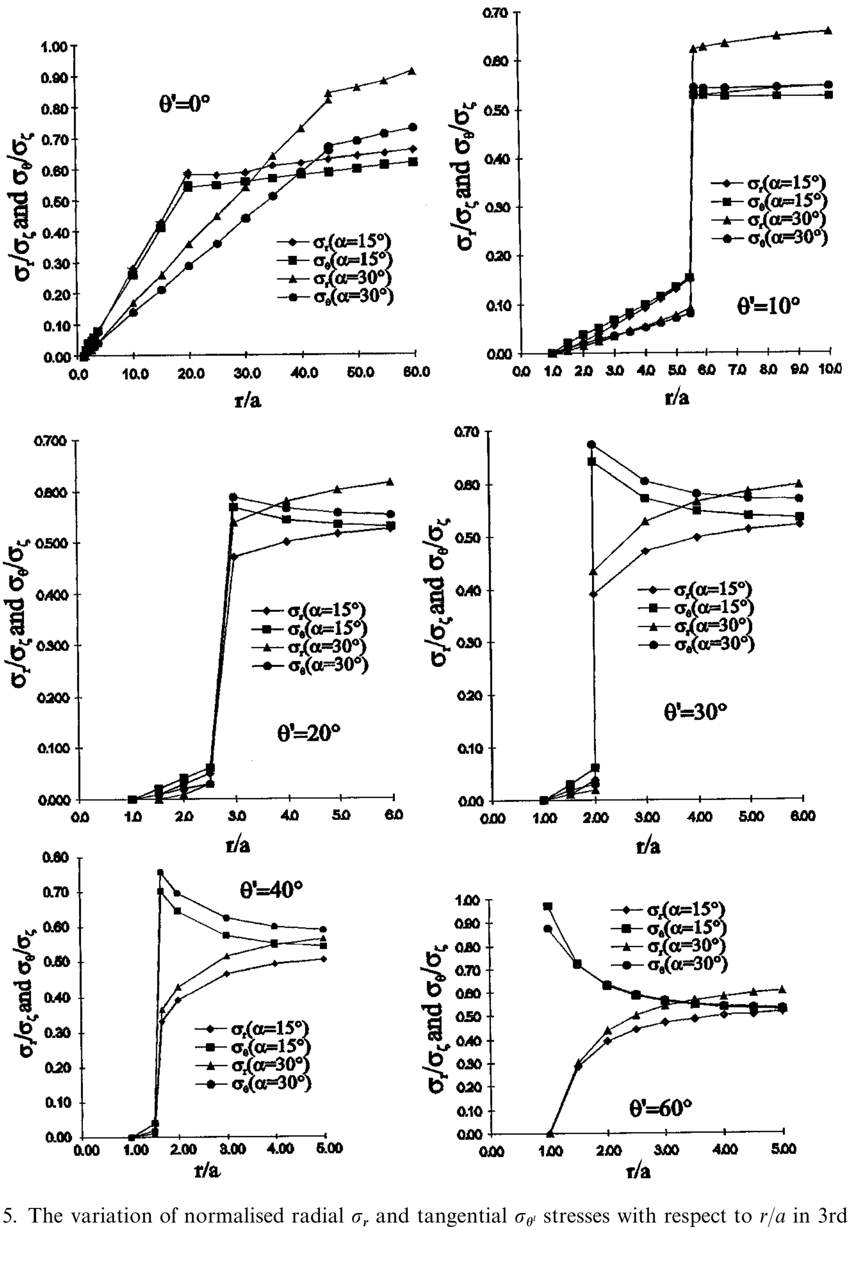



Figure 5. The variation of normalised radial $\sigma_r$ and tangential $\sigma_{\theta'}$ stresses with respect to $r/a$ in 3rd zone

$yy'$ axis towards the second zone, the radial stress distributions on the planes which are perpendicular to the axis of shaft with $\alpha = 15°$ and $\alpha = 30°$ inclination angles, respectively, tend to coincide to each other. This situation is reversed for $\sigma_\theta$ distributions. Generally speaking for the first zone, an increase in the inclination angle $\alpha$ does not induce significant change in the configuration of stress distribution.

The stress distribution in the third zone display remarkable features related with both the inclination angle $\alpha$ and the position or departure angle $\theta'$ (Figure 5). The intrrupted region existing in this zone alters the stress distributions. The geometry induces a linearly increasing overburden height from the boundary of shaft to the border of interrupted–uninterrupted regions. Consequently, the induced radial $\sigma_r$ and tangential $\sigma_\theta$ stresses are linerly increasing in this region. The overall stress distributions along the directions defined by departure angles $\theta'$ are systematically change by increasing $\theta'$. For instance, along the line defined by zero departure ($\theta' = 0$), the overburden and the induced radial $\sigma_r$ and tangential $\sigma_\theta$ stresses grow up to the values equal to those induced by undisturbed field stresses at the border of two regions. Whereas, the increase in departure with respect to $yy'$ axis makes the interrupted zone shortened. This phenomenon causes the sudden jumps of stresses at the border of two regions displayed by the plottings in Figure 5.

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